

Assignment 1: SQC & OR

Formulation of Linear Programming Problems



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# Question 1

This problem is known in the literature as ***Diet Problem***. Dieticians tell us that a balanced diet must contain quantities of nutrients such as fats, vitamins, minerals etc. The medical experts and dieticians tell us that it is necessary for an adult to consume at least 75 g of proteins, 85 g of fats, and 300 g of carbohydrate daily. Table 1 below gives the food items (which are readily available in the market), analysis and their respective cost.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Food**  **Type** | **Food Value (g) per 100 g** | | | **Cost per kg (Rs)** |
| **Proteins** | **Fats** | **Carbohydrates** |
| 1 | 8.0 | 1.5 | 35.0 | 1.00 |
| 2 | 18.0 | 15.0 | - | 3.00 |
| 3 | 16.0 | 4.0 | 7.0 | 4.00 |
| 4 | 4.0 | 20.0 | 2.5 | 2.00 |
| 5 | 5.0 | 8.0 | 40.0 | 1.50 |
| 6 | 2.5 | - | 25.0 | 3.00 |
| Minimum daily requirements | 75 | 85 | 300 |  |

Find out the food that should be recommended from a large number of alternative sources of these nutrients so that the total cost of food satisfying the minimum requirements of balanced diet is the lowest.

## Answer – Formulation of LP Problem

Let denote the amount of food intake of food type , where , calculated in the units of per 100 g.

As per given by the table above, the cost for consuming amount (in units of 100 g) of food of type 1 would cost rupees , since the cost of per kg of food of type 1 is 1.00 rupee. In a similar way, grams of food of type 2 would cost rupees, grams of food of type 3 would cost rupees, grams of food of type 4 would cost rupees, grams of food of type 5 would cost rupees and grams of food of type 6 would cost rupees.

Therefore, the total cost of consuming the food would be given by;

Now, according to the above table, we get to know that consuming 100 grams of food of type 1 would give 8 grams of proteins as nutrients. Hence, consuming grams of food of type 1 would give grams of proteins as nutrients. Similarly, consuming grams of food of type , would give and grams of proteins as nutrients respectively, where is equal to 2, 3, 4, 5 and 6.

Therefore, the total amount (in gram) of protein gained from the combination of food is;

According to dieticians, this amount of protein intake should be at least 75 grams daily. It introduces the constraint,

Similarly, considering the minimum daily requirements of Fats and Carbohydrates, we also obtain another two constraints;

, the constraint for minimum requirement of Fats and,

, the constraint for minimum requirement of Carbohydrate.

Also, since ’s denote the amount of food intake of type , it clearly must be non-negative.

The objective of the problem is to minimize the total cost of the food combination, i.e. minimizing the quantity . Therefore, the Linear Programming Problem can be formulated as follows;

**Minimize**

**Subject to the constraints;**

**And**

# Question 2

Two alloys, *A* and *B* are made from four different metals, I, II, III and IV, according to the following specifications:

|  |  |  |
| --- | --- | --- |
| Alloy | Specifications | Selling price ($)/ton |
| A | At most 80% of I | 200 |
| At least 30% of II |
| At least 50% of IV |
| B | Between 40% & 60% of II | 300 |
| At least 30% of III |
| At most 70% of IV |

The four metals, in turn, are extracted from three different ores with the following data;

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Ore | Max. Quantity (tons) | Constituents (%) | | | | | Purchase Price ($)/ton |
| **I** | **II** | **III** | **IV** | **others** |
| 1 | 1000 | 20 | 10 | 30 | 30 | 10 | 30 |
| 2 | 2000 | 10 | 20 | 30 | 30 | 10 | 40 |
| 3 | 3000 | 5 | 5 | 70 | 20 | 0 | 50 |

How much of each alloy should be produced to maximize the profit. Formulate the problem as a LP model.

## Answer – Formulation of LP Problem

Let and denote the amount (in ton) of produced alloys for alloy A and B respectively. Let and denote the amount (in ton) of the ores used of type 1, 2 and 3 respectively. Let denotes the amount (in ton) of -th Metal used to produce alloy A. Similarly, let denotes the amount (in ton) of the -th Metal used to produce alloy B. Here, can be any of .

It is assumed that the metals are extracted from the ores, and then they are mixed to form the alloys, rather than blending the ores together to form alloys which would cause the alloys to be comprised of other impurities in the ore. With this assumption in mind, the LP problem is formulated.

Selling tons of alloy A would yield a earning of rupees, while selling tons of alloy B would yield a earning of rupees. Also, using tons of ore of first type would rise to a cost of rupees and similar cost would be incurred for usage of other ores. Therefore, the profit is given by;

Now, based on the availability of the ores in tons as given in the table, we have the constraints; and .

Observe that, the total amount of metal I used to produce the alloys is . On the other hand, the total amount of metal I extracted from the ores is , since we get 20% of i.e. tons of metal I from ore of first type and so on. Obviously, the total amount of metal I extracted must be more than the total amount of metal I used to produce the alloys. Therefore, for metal I we have the constraint that;

We get three similar constraints for the other metals as well;

Also, the alloys should meet the required specifications. Therefore, for alloy A, the amount of metal 1 used to produce alloy A, i.e. to be less than or equal to 80% of the total amount of alloy A, i.e. . This give us the constraint; . Having at most 30% of alloy A being metal II, we get and at least 50% being metal IV gives . In addition to that, we must have the total amount of metal used to build up an alloy equal to the total amount of alloy produced. Therefore,

We can put this equality back into the objective function and the constraints to get rid of the variable . However, using simplifies our notation. Similar inequality and equality constraint comes up when dealing with specification requirements of alloy B.

The goal is to maximize the total amount of profit or . The Linear Programming problem can be formulated as follows;

Maximize

Subject to;

, ,

, , ,

and

**Note:**  is implied from the equality constraints and the non-negativity of ’s and ’s. Hence, they are excluded.